- 2 -
TRANSDUCER MECHANISMS

T-MATRIX MODELS

2.1 Moving Coil Motor Structure[1]

In our development of a basic understanding of how a transducer works, we will consider three fundamental parts: the motor, the electrical to mechanical conversion, the diaphragm mechanics, and the sound radiation. We will investigate these elements in this same order.

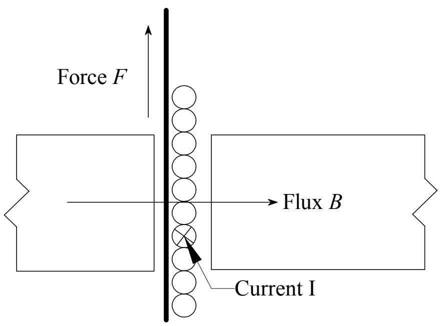

Figure 2-1 - Moving coil topology with definitions

Nearly all loudspeakers use the same basic motor structure, a coil of wire – the voice coil – immersed in a static magnetic field. There is a good reason for the predominance of this type of motor structure and we will see why. A simple voice coil configuration in axi-symmetric cross section, with the terms defined, is shown below. The flux can be created by any means necessary, although it is usually a permanent magnet. When a current I is sent through the coil (into the page) a force F is created as

$$F(\omega) = Bl\, I(\omega) \tag{2.1.1}$$

B = Flux density

l = length of wire in B

In order to completely define this object both the input (electrical) and the output (mechanical) must be considered. In other words, we must also consider the effect

1. See Beranek *Acoustics*

that this object has on the electrical side. We know that it will have a back EMF given by

$$E(\omega) = \frac{V(\omega)}{Bl} \tag{2.1.2}$$

These two equations can be conveniently combined into a single T-matrix form as

$$\begin{bmatrix} E(\omega) \\ I(\omega) \end{bmatrix} = \begin{bmatrix} 0 & Bl \\ 1/Bl & 0 \end{bmatrix} \begin{bmatrix} F(\omega) \\ v(\omega) \end{bmatrix} \tag{2.1.3}$$

Note that this formulation does not contain any restrictions on the actual displacement of the coil. However, in practice, the length of the coil winding limits the real excursion. The coil can, of course, be made to be of any length desired, although there are practical limits. Therefore, this type of motor has the capability of large excursions if one is willing to accept the loss of efficiency that is associated with allowing these excursions.

2.2 *Variable Reluctance Motor Structure*[2]

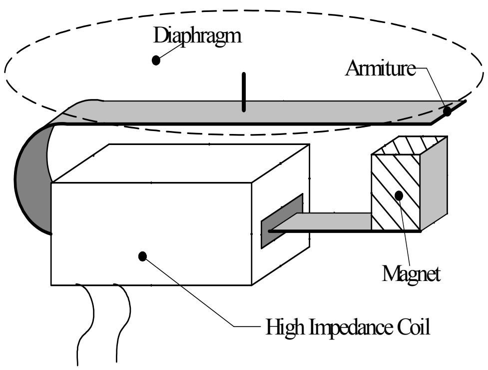

Figure 2-2 - Variable reluctance motor structure

The second most common form of motor structure is the variable reluctance motor. An example of which is shown in Fig. 2-2. This type of motor has wide application in hearing aids for several reasons. The most important is the ability to make it very small. The second is its high internal impedance. This motor can deliver a large force efficiently, but with only a very limited excursion capability. We will see these characteristics later in this chapter.

There is a flux path around the magnet material of the *armature* and back through the magnet. If a current is sent through the coil of wire then the flux will be modulated according to the current. This modulating flux will cause the armature to be attracted to and repelled from the magnet (relative to its static position) in direct proportional to the current in the coil (to first order). When the armatures motion is attached to a diaphragm then sound is emitted from the diaphragm.

2. See Hunt, *Electroacoustics.*

This motor can be analyzed as follows. Hunt shows the basic form for a magnetic gap as

$$E(\omega) = \left(z_e + \frac{i\omega N^2 \mu_0 S_g}{d} \right) I(\omega) + \frac{N B_g S_g}{d} V(\omega)$$
$$F(\omega) = \frac{N B_g S_g}{d} I(\omega) + \left(z_m + \frac{B_g^2 S_g}{i\omega d\, \mu_0} \right) V(\omega) \tag{2.2.4}$$

B_g = the gap flux,
Sg = the gap area,
d = the gap width,
N = number of turns in coil,
μ_0 = the permeability of free space

These variables are shown in Fig. 2-3.

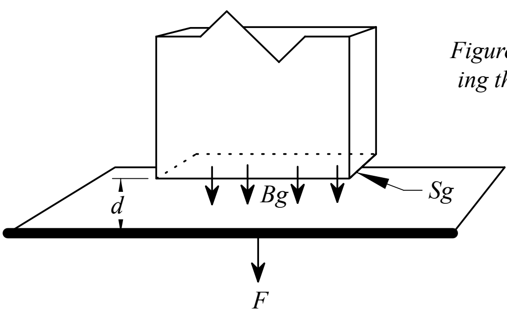

Figure 2-3 - Tractive air gap showing the variables used in the equa-

Rewriting this into a form which is more usable to us (electrical on one side, mechanical on the other) we obtain

$$\begin{bmatrix} E(\omega) \\ I(\omega) \end{bmatrix} = \begin{bmatrix} \dfrac{z_e + z_e'}{T} & \dfrac{z_m(z_e + z_e')}{T} - \dfrac{T z_e}{z_e'} \\ \dfrac{1}{T} & \dfrac{z_m}{T} - \dfrac{T}{z_e'} \end{bmatrix} \cdot \begin{bmatrix} F(\omega) \\ V(\omega) \end{bmatrix} \tag{2.2.5}$$

$$z_e' = i\omega \frac{N^2 \mu_0 S_g}{d}$$

$$T = \frac{N B_g S_g}{d}$$

The variable z_e represents all of the electric domain impedances of the coil that do not contribute to creating a force. This term is essentially the DC resistance of the coil, but there can also be other parasitics involved.

Eq. (2.2.5) is not in a very attractive form for our usage. It is hardly intuitive and not very useful as it is. With some (off-line) matrix manipulations we can find a more amenable form as shown below

$$\begin{bmatrix} E(\omega) \\ I(\omega) \end{bmatrix} = \begin{bmatrix} 1 & z_e + z_e' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & T \\ \dfrac{1}{T} & 0 \end{bmatrix} \begin{bmatrix} 1 & z_m - \dfrac{TB_g}{i\omega N \mu_0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F(\omega) \\ V(\omega) \end{bmatrix} \tag{2.2.6}$$

This form is nearly what we desire with the exception of the term being subtracted from the mechanical impedance of the system. This term has the form of a stiffness in the mechanical domain. It is in fact a true negative stiffness that is a direct result of this form of motor structure. If this negative stiffness becomes greater than the mechanical stiffness of the system then the system becomes unstable. The armature will lock up in a closed position, unable to return to a neutral point.

We still seek a more concise form for the motor structure alone without the complications of the electrical and mechanical impedances mixed in. With a little more manipulation we can rewrite Eq. (2.2.6) as

$$\begin{bmatrix} E(\omega) \\ I(\omega) \end{bmatrix} = \begin{bmatrix} 1 & z_e + z_e' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & T \\ \dfrac{1}{T} & \dfrac{T}{z_e'} \end{bmatrix} \begin{bmatrix} 1 & z_m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F(\omega) \\ V(\omega) \end{bmatrix} \tag{2.2.7}$$

While progressing, we are still not there. We would like to get each matrix to contain only one variable so that we can see how they link together. In order to do that we will need to write out the terms and pull out a matrix that depends on N. This will result in

$$\begin{bmatrix} 1 & z_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} N & 0 \\ 0 & {}^{1}\!/_{N} \end{bmatrix} \begin{bmatrix} 0 & \dfrac{i\omega\mu_0 S_g}{d} \\ \dfrac{-d}{i\omega\mu_0 S_g} & 1 \end{bmatrix} \begin{bmatrix} 0 & \dfrac{B_g S_g}{d} \\ \dfrac{d}{B_g S_g} & 0 \end{bmatrix} \tag{2.2.8}$$

We can begin to see that this structure works similarly to a voice coil, except that the coupling constant depends on the gap area and width, and there is a strange matrix just ahead of the coupling matrix. This matrix accounts for the interactions of the mechanical and electrical domains in a way that is not represented by a gyrator or a transformer. These terms can be thought of as time derivatives of the quantity $\mu_0 S_g / d$. It would be possible to further pull out the terms S_g / d into a stand alone matrix, but there does not seem to be any compelling reason to do that. One final step is instructive and that is to move the electrical matrix for the static impedance of the coil inside the turns matrix and define a new variable r_N to be the resistance per turn. Finally, multiplying the last two matrices together yields a simple, albeit still not very intuitive matrix result

$$\begin{bmatrix} N & 0 \\ 0 & 1/N \end{bmatrix} \begin{bmatrix} 1 & r_N \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{i\omega\mu_0}{B_g} & 0 \\ \dfrac{d}{B_g S_g} & \dfrac{-B_g}{i\omega\mu_0} \end{bmatrix} \tag{2.2.9}$$

This equation can also be written with the electrical impedance left out and the turns multiplied out to yield an alternate result for the coupling matrix

$$\begin{bmatrix} \dfrac{i\omega N\mu_0}{B_g} & 0 \\ \dfrac{d}{NB_g S_g} & \dfrac{B_g}{i\omega N\mu_0} \end{bmatrix} \tag{2.2.10}$$

A few points are worth noting. First, this form is closer to that of a transformer than a gyrator, although it is neither. The coupling from the voltage to the force decreases with increasing *N* and decreasing *B*. The current to force increases with both. For large *N*, as is most common, the off diagonal term is small. This type of motor structure becomes more efficient with voltage drive at higher frequencies. It is a high impedance device because of the large *N* of typical designs and tends to couple well into systems that require large forces with small excursions. This structure might be useful for tweeters, but not for woofers. It is very effective for hearing aid transducers which see a high mechanical load compared to free space.

In practice, one wants to have the static flux and the dynamic flux follow separate paths to avoid flux modulation and distortion. There are many ways to do this and actual implementations are left to further reading. One must also not forget that the gap width is modified by the permeability of the magnet if the magnet is placed adjacent to the gap. This virtual gap, not the physical gap, is greater than the actual gap as a result of this permeability.

2.3 Magnetic Circuits[3]

In both of the previous motor structures, a permanent magnet is usually used to supply the static energy for actuation. In this section, we will develop the basic equations for designing a motor structure.

The simplest way to think about a motor structure is as a closed circuit, much like an electric circuit. Given such a circuit, shown in Fig. 2-4, the following equation must hold for continuity of energy

3. See Parker, *Design and Analysis of Permanent Magnet Structures.*

$$L_m H_m - L_g H_g = 0 \tag{2.3.11}$$

L_m = *the magnet length*

H_m = *the magnetizing potential per unit length*

L_g = *the air gap length (width in later discussions)*

H_g = *the air gap potential (which must be numerically equal to the air gap flux density* B_g*)*

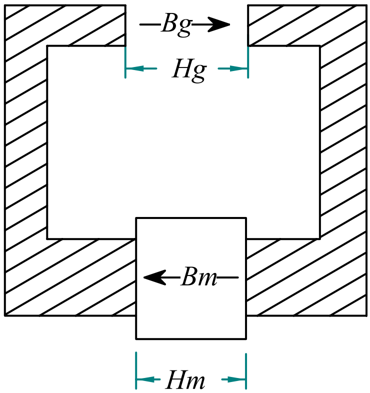

Figure 2-4 - Magnetic circuit definitions

We have assumed that there is no potential loss within the connecting arms of this circuit, which is a reasonable assumption if the material is of good magnetic permeability. If we further assume, again incorrectly, but a reasonable first cut assumption, that flux is conserved in this circuit then

$$A_m B_m = A_g B_g \tag{2.3.12}$$

A_m = *the magnet area,*

B_m = *the magnets flux density at its operating point and*

A_g = *the gap area*

From these equations, we can derive the fundamental set of design equations for the magnetic circuit given the desired gap properties

$$L_m = \frac{L_g H_g}{H_m}$$
$$A_m = \frac{A_g B_g}{B_m} \tag{2.3.13}$$

These equations require that the operating point of the magnet is known. The operating point is the intersection of the curve representing the magnets flux versus magnetization potential and the load line. In detail, this determination is a nonlinear process, but in practice we simply assume its location. Fig. 2-5 shows a typical magnetic property chart for a common ferrite magnet. We want to operate

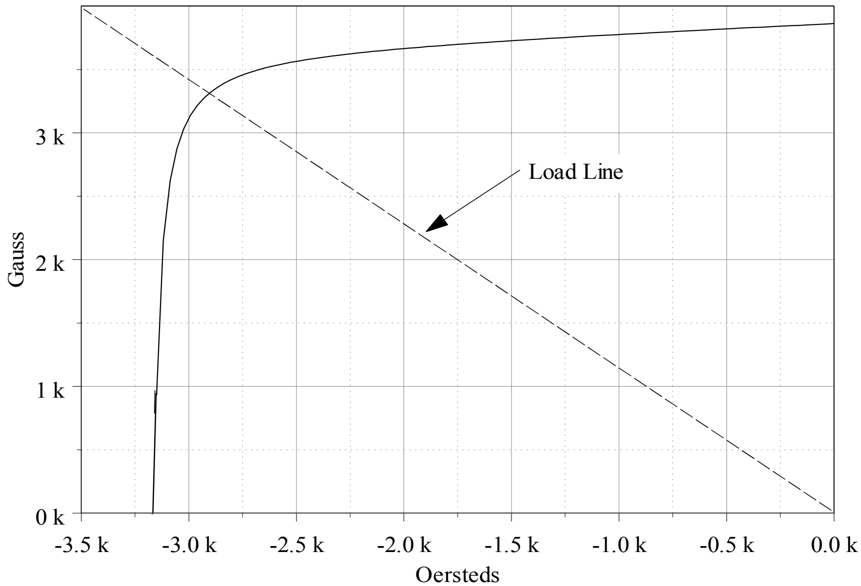

Figure 2-5 - Ceramic magnet load line calculation - the dotted line is the load line

a magnet at the knee of the curve where the energy product is at its maximum. This minimizes the amount of magnet material required to perform the task. From the figure, we can see that the ratio of B_m to H_m should be about 1.2 (ignoring the minus sign). This is the value of the load line L_l that is required for optimum magnet usage. This value will differ for each magnet type, but otherwise the rest of the analysis in this section would be identical for all magnet types. From Eq. (2.3.12) and Eq. (2.3.13) we know that

$$\frac{B_m}{H_m} = \frac{L_m A_g}{A_m L_g} = L_l \tag{2.3.14}$$

and from this it follows directly that

$$\frac{A_m}{L_m} = \frac{A_g}{L_l \cdot L_g} \tag{2.3.15}$$

This equation determines the aspect ratio of the magnet. By once again using Eq. (2.3.14) we can readily determine the area of the magnet as

$$A_m = A_g \frac{B_g}{B_m} \tag{2.3.16}$$

thus completing the design.

This approach will yield a good estimate of the magnet required for a particular design, but it is an estimate that almost always yields a magnet that is too small. Several factors enter into the problems which confound the design:

- The first discrepancy between theory and an actual design is that there can be a great deal of leakage flux, which depends greatly on the geometry of the motor design. Flux leakage can be as high as 20% or more (in a poorly designed motor) or as low as 5% in a well-designed structure. Larger magnet volumes, like ceramic, do not lend themselves to motor structures which are very efficient yielding typically 10% – 20% loss. Smaller magnet volumes, like Neodymium, and rare earth magnets, can be made to be quite efficient on the order of 5%-10% loss.
- There will also be some potential loss in the motor structure material that depends on the actual material selected.
- The magnetic material of the motor is not linear and its loss depends on the details of the flux flow pattern.

The detailed design of a motor structure can involve a numerical calculation of the actual flux patterns and the leakage flux. In our experience, these calculations are only warranted in cases where one is concerned with saving a few percent on the cost of the motor structure. For very high volume production, this can be worthwhile but for mid to low volume production simply over-designing the motor structure will yield more than satisfactory results. Typical numerical analysis of a well-designed structure will yield up to 10% improvements in material costs depending on the level of optimization of the original design.

Following a few simple rules of thumb one can get very close to the required design:

- There will always be flux loss, a value of 5%-20% (as given above) should be expected so design for more gap flux than you need.
- Minimize saturation points. In a moving coil structure this is almost always at the base of the pole piece where it joins the backplate. Saturation causes a lower amount of flux in the gap. It is good to saturate the gap at the inner radius as this is the point where the greatest flux should occur and saturation at this point minimizes flux modulation effects from the moving coil.
- Make the gap as symmetric as possible with cone excursion. This minimizes distortion and reduces cone displacement out of the gap at high output levels.
- Contrary to popular marketing of loudspeakers, the mass of the magnet is unimportant; it's the flux in the gap that counts.

In conclusion, the design of the motor structure need not be a complicated task. There are very few sound quality issues associated with the design of the motor structure as long as one abides by the rules above. The principle design issues are mostly cosmetic and financial.

2.4 The Electrostatic Motor[4]

The electrostatic motor is just about as old as the moving coil. We will develop its T-matrix forms in this chapter and use it in a later chapter when we talk about microphones. Another common usage of this motor is for loudspeakers. We will not get into that application, as it is not really that common. A direct comparison between the moving coil and the electrostatic motors would show the electrostatic to be less efficient. It is also inherently nonlinear and has a relatively low excursion capability. The interested reader could easily show this to themselves using the techniques developed at the end of this chapter.

In the electrostatic motor there is a diaphragm, which is free to move, usually a stretched membrane, and a fixed backplate which make up a capacitor. (As a side note, the modal characteristics of the stretched membrane of an electrostatic loudspeaker are the reason that we will not discuss it here. To do so would require an excursion into the field of mechanical vibrations. We have chosen not to discuss this topic in this text because it is already well covered in other texts. Leaving out the mechanical vibrations from a discussion of electrostatic loudspeakers would do it a serious injustice.)

The membrane and backplate capacitor are charged with an external voltage E. This polarizing voltage can come from any source but needs to be extremely clean. Typical voltages range from several hundreds to several thousand volts. Either the diaphragm or the backplate is usually grounded and the other plate is coupled to ground through a very large resistance. This motor can be used in two ways: as a receiver, or a source. In the source case, the polarizing voltage is modulated and the variable force on the plate causes the membrane diaphragm to move. In the receiver case the moving diaphragm causes a current to flow through the resistor load because the voltage is fixed and the capacitance changes. Since we will be using this motor in its receiver configuration, we will derive it in that form. However, simply taking its inverse will put it in the form for use as a source.

In the receiver mode the following equations must hold

$$C_0 = \frac{Q}{E} \tag{2.4.17}$$

$Q = total\ charge\ on\ capacitor\ plates$
$E = polarizing\ voltage$
$C_0 = diaphragm\ capacitance$

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4. See Hunt, *Electroacoustics*

As the diaphragm moves the capacitance changes and since the voltage is constant the charge on the plates must vary. This leads to the relation

$$C_0'(t) = C_0 + C_e(t) = \frac{\varepsilon_0 S}{d - x(t)} \tag{2.4.18}$$

$C_0'(t) = \textit{time dependent capacitance}$
$C_0 = \textit{static capacitance}$
$C_e(t) = \textit{variable capacitance}$
$\varepsilon_0 = \textit{permittivity of air,}$
$S = \textit{effective area of backplate (assuming the backplate} < \textit{diaphragm)}$
$d = \textit{air gap}$
$x(t) = \textit{diaphragm displacement}$

For small displacements this equation can be "linearized" (the source of the inherent nonlinearity) as

$$C_0'(t) \cong C_0\left[1 + \frac{x(t)}{d}\right] \tag{2.4.19}$$

We will need the time dependent form of Eq. (2.4.17)

$$Q(t) = q_0 + q(t) = C_0'(t)\left[E + e(t)\right] \tag{2.4.20}$$

$q_0 = \textit{static charge on plates}$
$q(t) = \textit{dynamic charge on plates}$
$E_0 = \textit{polarizing voltage}$
$e(t) = \textit{dynamic voltage across plates}$

So that

$$q(t) = C_0 E_0 + C_0 e(t) + C_0 E_0 \frac{x(t)}{d} + C_0 \frac{x(t)}{d} e(t) \tag{2.4.21}$$

The last term is not linear and it is small (another nonlinearity) so we will ignore it. Taking the time derivative of this equation results in

$$I(\omega) = i\omega C_0 E_0 + i\omega C_0 E(\omega) + \frac{C_0 E_0}{d} V(\omega) \tag{2.4.22}$$

where we should recognize the first of the two equations that we need for our T-matrix. Obviously the other one must involve the force.

We know that the force between the plates of our capacitor must be the mechanical impedance times the velocity (in the frequency domain) plus a variable force due to the changing charge on the plates. This can readily be seen as

$$f(t) = z_m v(t) + \frac{E_0}{d} q(t) \tag{2.4.23}$$

$z_m = \textit{mechanical impedance of the diaphragm}$

(We have taken the liberty of mixing domains here since there really is no such thing as a time domain impedance.) Taking the time derivative gives us the second equation (in the frequency domain)

$$F(\omega) = z_m V(\omega) + \frac{E_0}{i\omega d} I(\omega) \tag{2.4.24}$$

We can now write the coupling T-matrix, but we need to note one other thing. Since our input is a pressure and our output an electric signal we will want to reverse our usual definitions as follows

$$\begin{pmatrix} F(\omega) \\ V(\omega) \end{pmatrix} = \begin{pmatrix} -i\omega z_m \dfrac{d}{E_0} & \dfrac{E_0}{i\omega d} + z_m \dfrac{d}{C_0 E_0} \\ -i\omega \dfrac{d}{E_0} & \dfrac{d}{C_0 E_0} \end{pmatrix} \begin{pmatrix} E(\omega) \\ I(\omega) \end{pmatrix} \tag{2.4.25}$$

We can immediately separate out the electrical capacitance of the diaphragm as

$$\begin{pmatrix} F(\omega) \\ V(\omega) \end{pmatrix} = \begin{pmatrix} C_0 \dfrac{E_0}{d} & \dfrac{E_0}{i\omega d} + z_m \dfrac{d}{C_0 E_0} \\ 0 & \dfrac{d}{C_0 E_0} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i\omega C_0 & 1 \end{pmatrix} \begin{pmatrix} E(\omega) \\ I(\omega) \end{pmatrix} \tag{2.4.26}$$

and then separate off the mechanical impedance as

$$\begin{pmatrix} F(\omega) \\ V(\omega) \end{pmatrix} = \begin{pmatrix} 1 & z_m \\ 0 & 1 \end{pmatrix} \begin{pmatrix} C_0 \dfrac{E_0}{d} & \dfrac{E_0}{-i\omega d} \\ 0 & \dfrac{d}{C_0 E_0} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i\omega C_0 & 1 \end{pmatrix} \begin{pmatrix} E(\omega) \\ I(\omega) \end{pmatrix} \tag{2.4.27}$$

There is yet another way to view this matrix combination, as shown below.

$$\begin{pmatrix} F(\omega) \\ V(\omega) \end{pmatrix} = \begin{pmatrix} 1 & z_m + \dfrac{1}{i\omega C_{me}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} C_0 \dfrac{E_0}{d} & 0 \\ 0 & \dfrac{d}{C_0 E_0} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i\omega C_0 & 1 \end{pmatrix} \begin{pmatrix} E(\omega) \\ I(\omega) \end{pmatrix} \tag{2.4.28}$$

This equation has, perhaps, a more elegant look. We can use either form so long as we remember to add the negative compliance which results from the electrodynamic coupling to the diaphragm's mechanical impedance.

2.5 *The Piezoelectric Motor Structure*[5,6]

The final motor structure that we will consider, only briefly, is the piezoelectric motor. In general, these motors can be extremely complex owing to the fact

5. See Kinsler, *Fundamentals of Acoustics*
6. See Morse, *Vibration and Sound*

that the complete solution involves a 3 X 3 tensor. If we limit ourselves to the situation shown in Fig. 2-6, then both Kinsler, et al. and Morse show equations that eventually lead to identical results. This derivation follows a mixture of the two.

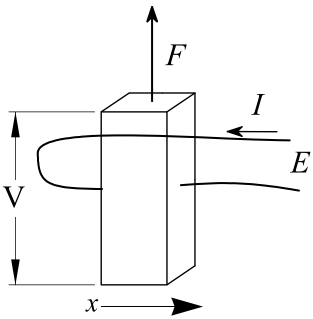

Figure 2-6 - Diagram of piezoelectric X-cut crystal actuator

From Kinsler and Frey

$$\sigma = \frac{\varepsilon_x \varepsilon_0 E}{l} - d_{12}\left(\frac{F}{S}\right) \tag{2.5.29}$$

and

$$\frac{\partial \eta}{\partial y} = -s_{22}\left(\frac{F}{S}\right) + d_{12}\,\frac{E}{l} \tag{2.5.30}$$

σ = *the charge density on the x faces,*

η = *the strain in the direction of the force,*

d_{12}, s_{22} = *material properties.*

Converting these equations into ones in current and velocity yields

$$\begin{aligned} I &= i\omega \frac{\varepsilon_x \varepsilon_0\, S}{l} E - i\omega\, d_{12} \cdot F \\ V &= -s_{22}\left(\frac{l}{S}\right) F + d_{12}\, E \end{aligned} \tag{2.5.31}$$

Rewriting in standard form we get

$$\begin{bmatrix} \tau & \dfrac{1}{i\omega C_p \tau} \\ -i\omega C_0 \tau & \left(\dfrac{C_0}{C_p} + 1\right)\tau^{-1} \end{bmatrix} \tag{2.5.32}$$

$$C_0 = \left(\frac{1}{k^2} - 1\right) \cdot C_p, \quad C_p = \frac{l_y l_z d_{12}^2}{s_{22} l_x}, \quad \tau = \frac{s_{22}}{d_{12} l_z}.$$

This equation can also be written as

$$\begin{bmatrix} 1 & 0 \\ i\omega C_0 & 1 \end{bmatrix} \begin{bmatrix} \tau & 0 \\ 0 & \tau^{-1} \end{bmatrix} \begin{bmatrix} 1 & \left(i\omega C_p \right)^{-1} \\ 0 & 1 \end{bmatrix} \tag{2.5.33}$$

The above equation shows the equivalent circuit to be an input capacitor C_0 in parallel with a transformer of turns ratio τ, followed by a series capacitor C_p. For our purposes, the T-matrix shown in Eq. (2.5.32) is more convenient since the capacitive elements are an integral part of the motor and should not be considered as separate elements.

Many forms for a piezoelectric motor are possible and the one given above is just an example of how one puts a given set of equations into the form of a T-matrix. Typical values for the material constants are given by Kinsler for a quartz crystal. These are the values that are used in the examples here.

2.6 A Simply Supported Piston

The T-matrix for a simply supported piston, like a loudspeaker cone when it moves as a rigid body, is very simple. The impedance of this device can be derived from Newton's Equation written in the frequency domain and in terms of velocity

$$F(\omega) = i\omega M_m V(\omega) + R_m V(\omega) + \frac{1}{i\omega C_m} V(\omega) \tag{2.6.34}$$

M_m = the mechanical mass

R_m = the mechanical resistance

C_m = the mechanical compliance

The subscript *m* refers to mechanical elements.

The impedance function then becomes

$$z_m(\omega) = \frac{F(\omega)}{V(\omega)} = i\omega M_m + R_m + \frac{1}{i\omega C_m} \tag{2.6.35}$$

And finally the T-matrix is

$$\begin{bmatrix} 1 & i\omega M_m + R_m + \dfrac{1}{i\omega C_m} \\ 0 & 1 \end{bmatrix} \tag{2.6.36}$$

2.7 A Simple Driven Piston Example

In order to show the power of the above formulations and the T-matrix approach, we will consider a variation of the problem shown in section 1.10. We will derive the motion of the diaphragm as a function of frequency for a constant one volt input for the three different motor structures.

The matrices, when laid out, are

$$\begin{bmatrix} E(\omega) \\ I(\omega) \end{bmatrix} = \begin{bmatrix} 1 & z_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Motor \\ Structure \end{bmatrix} \begin{bmatrix} 1 & i\omega M_m + R_m + \dfrac{1}{i\omega C_m} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F(\omega) \\ V(\omega) \end{bmatrix} \tag{2.7.37}$$

We will look at two cases. First, we will assume that there are no additional forces acting on the cone for it is in a vacuum, and, second, that a large load, a heavy fluid like water, is placed on the piston.

Multiplying out the matrices for the moving coil case, we obtain

$$\begin{bmatrix} E_{in} \\ I(\omega) \end{bmatrix} = \begin{bmatrix} \dfrac{R_e}{Bl} & \dfrac{R_e\left(i\omega M_m + R_m + \dfrac{1}{i\omega C_m}\right)}{Bl} + Bl \\ \dfrac{1}{Bl} & \dfrac{i\omega M_m + R_m + \dfrac{1}{i\omega C_m}}{Bl} \end{bmatrix} \begin{bmatrix} F(\omega) \\ V(\omega) \end{bmatrix} \tag{2.7.38}$$

Further multiplying out this equation and looking only at the term for the voltage (the top equation) we will get

$$E_{in} = \frac{R_e}{Bl} F(\omega) + \left(\frac{R_e}{Bl} \left(i\omega M_m + R_m + \frac{1}{i\omega C_m} \right) + Bl \right) V(\omega) \tag{2.7.39}$$

from which we can see that

$$V(\omega) = \frac{E_{in} Bl}{R_e Z(\omega) + R_e \left(i\omega M_m + R_m + \dfrac{1}{i\omega C_m} \right) + Bl^2} \tag{2.7.40}$$

$$Z(\omega) = \frac{F(\omega)}{V(\omega)} = \textit{an external mechanical impedance.}$$

Similar equations can be developed for the variable reluctance motor as well as the piezoelectric motor structures. This exercise is left for the reader.

The two graphs shown in Fig. 2-7 are curves of the pressure responses (arbitrary scale) for the moving coil (MC) and variable reluctance (VR) motor structures driving a typical diaphragm ($\omega_s \approx 100\,\mathrm{s}^{-1}$, mass = 15g). The moving coil is $8\,\Omega$ with a $Bl = 5.0$ Nt./A. The variable reluctance has a gap flux B_g of .8 Tesla (8000 Gauss) and a magnetic gap area of .01 m². The voltage drive is one volt and the current drive is 12 mA.

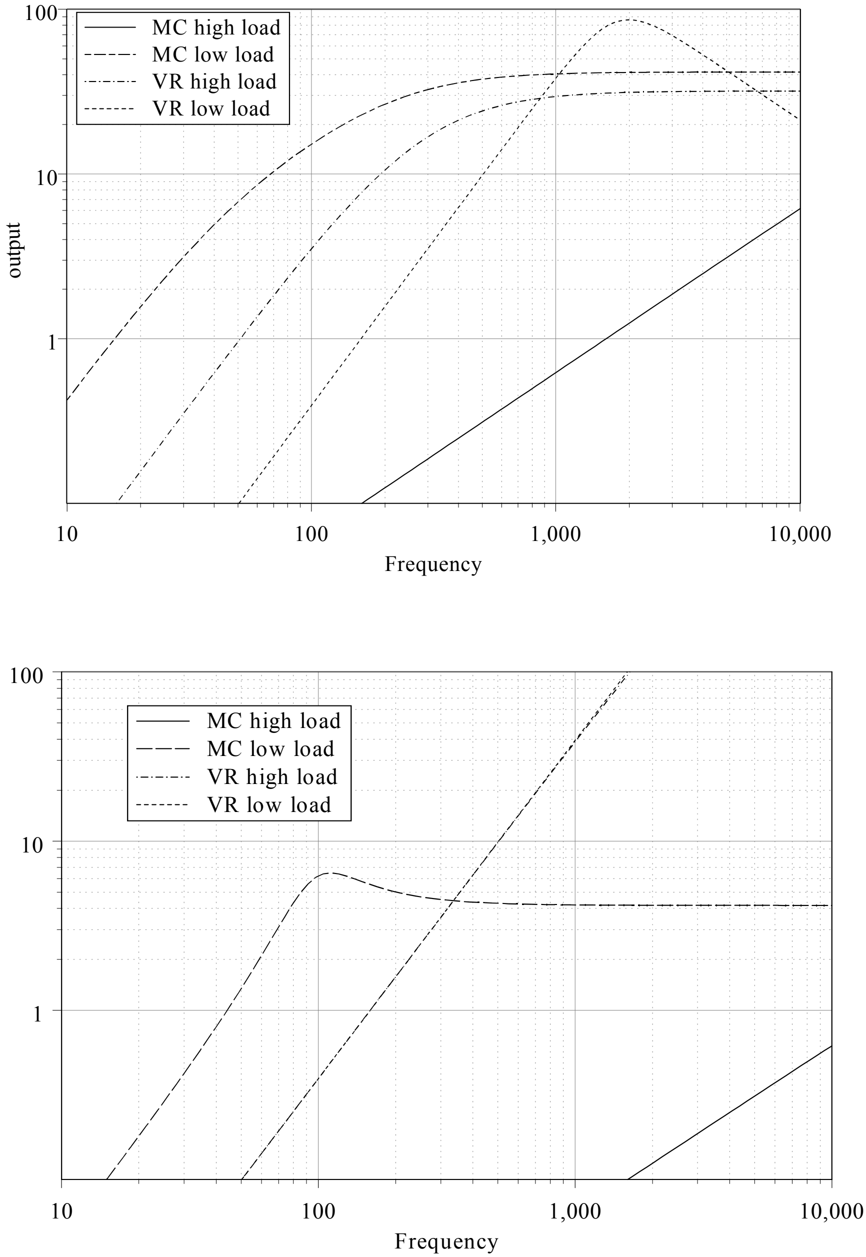

Figure 2-7 - Moving coil and variable reluctance motor structures driving a mechanical load with voltage (top) and current (bottom) drives

The moving coil is clearly the better choice for the lightly loaded condition for either of the source types. However, at higher loads the variable reluctance device looks attractive with either source. The variable reluctance motor is independent of the source configuration within the frequency range shown. Air loads are very small under most circumstances, indicating the reason for the predominance of the moving coil motor structure in the design of loudspeakers. The piezoelectric motor is six orders of magnitude below the curves shown above. It is not even worth considering as far as efficiency is concerned, at least not for theses loads in this frequency range.

There always seems to be a new concept in motor structures, but seldom have they made so much as a dent in the usage of the moving coil. In any event, the techniques shown in this section can always be used to determine the effectiveness of any new motor concept in any configuration.

2.8 Summary

We have seen how the T-matrix approach is very effective in analyzing a wide variety of motor structures and we will continue to see its usefulness throughout this text. The output (displacement, velocity and pressure, or voltage and current) for almost any form of motor and cone assembly whether driving a fluid or being driven by it, under any type of load and driven with any type of source can be easily assessed with these techniques. Once we know how to convert a diaphragm velocity into a sound field, or visa-versa, the problem of converting between electrical signals and sound will be done. When we consider the enclosure problem using the same T-matrix formulation that we have seen here, we will see that it is just as effective a technique in that situation. The wide variety of enclosure types in common usage will make the T-matrix approach even more appealing. And finally, this approach is a virtual requirement for the accurate calculation of waveguide characteristics.

The T-matrix approach is particularly well suited to the problem of describing the *motor* of a transducer. Any form of energy conversion (motor) can be defined in a straightforward manner. These components can then be dropped into a mechanical structure in a very efficient manner to yield and analysis of virtually any type of transducer.